STEADY, GRADUALLY VARIED FLOW IN UNIFORM CHANNELS ON MILD SLOPES

BY

MING LEE

B.S., Massachusetts Institute of Technology, 1943

M.S., Harvard University 1944

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE SCHOOL OF THE
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The laboratory apparatus described herein was built in conjunction with the experimental investigation on the Hydraulic Jump, conducted by Mr. E. R. Baumann.

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Steady, Gradually Varied Flow in Uniform Channels on Mild Slopes

A. Purpose and Scope of Research:

The purpose of the research is to develop a method of computing surface profiles of steady, gradually varied flow in uniform channels whereby the results can be obtained more directly than are possible hitherto with the existing methods. In order to determine the accuracy of the proposed method, actual profiles of flow were recorded in experimental channels of rectangular, trapezoidal and circular cross-sections under various conditions of flow. The tests data were compared with the computed results by the proposed method, and also by the other methods that are in use. The investigation is confined mainly to the back-water and drop-down curves in uniform channels on mild slopes inasmuch as they form the more important classes of surface profiles encountered in practice.

B. Derivation of the Gradually Varied Flow Equation:

The basic assumptions leading to the equation of gradually varied flow are discussed at some length. These simplifying assumptions are based upon the one-dimensional method of analysis, in which the flow is considered as being essentially parallel. The differential equation for gradually varied flow is derived from the simplified equation of momentum, as contrast to the more common procedure of deriving it from energy considerations. The difference between the two forms of the equation is explained. It is concluded that the form derived from the momentum principle is more consistent with the empirical expressions for the evaluation of the friction slope since the latter expressions are essentially momentum equations[1].

C. Methods of Solving the Gradually Varied Flow Equation:

(i) Existing Methods.

The solution of the differential equation for gradually varied flow by graphical, approximate and direct integration is discussed with regard to the advantages and limitations of each method. The choice of the method to be used is decided by the type of problem under investigation. Theoretically, the graphical method[2] should yield results of the greatest accuracy inasmuch as no assumption is made in the integration procedure. The work involved in this method, however, is quite tedious so that it has not been widely used. In many practical problems the method of approximate integration or the Step Method[3]

may be used with sufficient accuracy. The limitation of the Step Method is that the computations must be made by small increments of depth in order to fulfill the assumptions made in evaluating the mean friction slope within each interval. The objection is not serious, however, in solving the type of problems in which only a few surface curves have to be computed and elevations are needed all along the channel. In another type of problem, on the other hand, such as the determination of the delivery in a channel connecting two reservoirs, a large number of surface curves is required under a wide variation of discharge. Under these circumstances, the use of the direct integration methods will prove to be of decided advantage in the time and labor of computations saved.

The progress of the various direct integration method represents the trend in developing a solution of the gradually varied flow equation to dispense with the large number of intermediate steps required by approximate integration. The earlier attempts[4] are incomplete in that either the effect due to velocity changes is neglected or the friction slope is evaluated by means of the Chezy equation with a constant coefficient, the use of which has since been outmoded. Moreover, these methods are based on some idealized channel cross-section of simple geometric shape. These simplifications may lead to a substantial deviation between the computed and observed profiles, as shown by the experiments conducted by Mononobe[4]. For closer agreement between the theoretical and actual results, the actual shape of the channel must be considered, and the friction slope should be evaluated by a more suitable friction formula than that of Chezy.

By using an empirical relationship between the "Conveyance" and the depth, Bakhmeteff[6] was able to derive a more general solution in which the effect due to the shape of the channel may be taken into consideration, and the friction slope may be evaluated by any suitable friction formula. His method is convenient when applied to problems in which the effect of velocity head changes is insignificant. Otherwise, the correction that must be made to take the velocity head changes into account necessarily destroys the convenience of his method. This objection is avoided in the method of Mononobe[4]; unfortunately it is marred by the difficulty in reading his graphical charts and the inherent approximations that are likely to result due to the error of interpolating the varied flow functions.

(ii) Proposed Method.

The proposed method is essentially an extension of Bakhmeteff's method to cover those cases involving significant effect due to velocity

head changes. The integral, embodying these changes, is reduced to a special form on the assumptions that both the conveyance and the area of a cross-section can be expressed as monomial functions of the depth. The values of the integral have been computed within a wide range, sufficient to meet the conditions encountered in most practical profile computations. The application and convenience of the proposed method are illustrated by solving several typical examples.

D. Laboratory Investigations:

In the laboratory actual surface profiles of flow were recorded in various experimental channels to determine the accuracy of the proposed method and the other existing methods in common use. The experiments were conducted under a closed system of circulation. Two 2-inch centrifugal pumps were used to lift water from a sump into a constant-head tank, from which the water flowed by gravity into a weir box through a 4-inch connecting pipe. The rate of flow was controlled by means of a 4-inch gate valve, the excess water being returned to the sump from the constant-head tank through a 3-inch drain.

The discharge used in each test was measured by the head on a 90-degree V-notch weir in the weir box, from which the water fell freely into an entrance tank. A hanging baffle was provided to cut down the turbulence at the inlet to the experimental channel. The depth of flow in the channel was controlled by adjusting the opening of the tail-water gate in the outlet box. Thence, the water returned to the sump, thus completing the circuit.

Experimental channels of rectangular, trapezoidal and circular cross-sections were used in the tests. The rectangular channel was made of 14-gage black iron, bent into 8-inch by 8-inch sides and width; the trapezoidal channel was made from the same channel by bending the vertical sides 70 degrees with the horizontal. These channels were fabricated in lengths of 4-feet and 8-feet. The joints of the separate sections were made water-tight by caulking them with plasticine clay. The circular channel was made of 24-gage black iron in 10-ft. sections. It was rolled by hand into a frame work to form the semi-circular section, approximately 10 inches in diameter. The various channels rested on 2-inch by 6-inch beams which were supported by screw jacks. By adjusting these jacks the channels could be set on the desired slope. The total length of the various channels was 40 ft.; however, to avoid the error caused by the turbulence at the inlet and outlet of the channel, only about 24 feet were used in observing the surface profiles in the tests.

The experimental procedure used in determining the surface curves followed essentially the same technique introduced by Mononobe[4]. That

is, the entire surface curve was obtained by conducting a series of profile determinations, the initial depth in a new run being adjusted approximately the final depth in the run before. By connecting the series of profiles thus obtained, it was possible to extend the total length of the curve beyond the limit confined to the effective length of the experimental channels. The applicability of this procedure has been well substantiated by tests conducted by Mitchell and Barron[6].

In the profile determinations the channels were all set on a slope of 0.001. Both the M_1 type (back-water) and the M_2 type (drop-down) surface curves were recorded for each channel. This was done by measuring the depths of flow by means of point gages at known distances along the channel. Each curve was obtained only under the rate of flow most suitable under the given conditions. The different rates of discharge used in the experiments ranged between 0.176 cfs. and 0.425 cfs.

The observed profiles are compared with the theoretical curves computed by the proposed, graphical, Step and Bakhmeteff's methods. The method of Mononobe is omitted because his graphical charts are not adequate for computing short sections as in the observed profiles. For this purpose the charts should be redrawn to such a scale as to be compatible with the required degree of accuracy of results. This has not been done inasmuch as the original values of the varied flow functions for the preparation of the charts are not included in Mononobe's paper.

E. Conclusions:

From the comparison of the test data and the results computed by the various methods, the following conclusions appear to be justified:

(i) that no substantial error is due to the assumptions made in the varied flow equation as indicated by the close agreement between the observed profiles and those computed by the graphical method.

(ii) that the surface curves computed by the proposed method agree reasonably well with the observed profiles so that it may be expected to apply to practical computations with sufficient accuracy.

(iii) that the computations of surface curves by the graphical, Step, proposed and Bakhmeteff's methods yield only slightly different results, the order of accuracy being as indicated.

It should be emphasized that the greater precision in the results of the graphical and Step methods is due to the fact that the computa-

tions are confined to small steps, for which the proposed method tends to be less accurate. However, the results of the tests indicate that the accuracy of the proposed method falls within the degree of refinement warranted in most practical profile computations. In practice, the proposed method will be decidedly more convenient to use inasmuch as the computations need not proceed by successive steps required in the other methods.

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Vita

Ming Lee was born in Hong Kong, China, on September 17, 1922. After receiving his early education under private tutoring, he attended the Diocesan Boys' School, and later King's College, from which he matriculated in the summer of 1939. He was enrolled in the School of Engineering at Hong Kong University until the fall of 1941 when he left for the United States to continue his studies. He completed his undergraduate work at Massachusetts Institute of Technology, receiving the degree of Bachelor of Science in 1943, and subsequently attended the Graduate School of Engineering at Harvard University. He was awarded the degree of Master of Science in Sanitary Engineering from Harvard University the following summer.

The author came to the University of Illinois as a half-time graduate research assistant in Civil Engineering in the fall of 1944. After being successful in passing the preliminary examinations for the doctorate in May, 1946, he was later appointed a Fellow in the same department. It was during the term of the latter appointment that the experimental investigations described in this paper were completed.

A progress report of the research conducted during his earlier appointment was presented to the 19th Annual Ohio Conference on Sewage Treatment in a paper entitled: "Treatment of Supernatant Liquor From Separate Digesters." A subsequent paper, "Constant Velocity Grit Chamber with Parshall Flume Control", written in collaboration with Professor H. E. Babbitt, was published in the Sewage Works Journal, Vol. XVIII, No. 4, July, 1946.

The author is a member in the Sigma Xi, Phi Kappa Phi, Delta Omega, Pi Mu Epsilon, and Mu San honorary societies.